SUPPLY-PRICE RELATIONSHIPS

FOR TEXAS GRAPEFRUIT

Confidential Report
to the
Executive Committee
Texas Valley Citrus Committee
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from the

Texas Agricultural Market Research and Development Center
Department of Agricultural Economics and Sociology
Texas A&M University

THE TEXAS AGRICULTURAL MARKET RESEARCH AND DEVELOPMENT CENTER

The purpose of the Center is to be of service to agricultural producers, groups and organizations, as well as processing and marketing firms in the solution of present and emerging marketing problems. Emphasis is given to research and educational activities designed to improve and expand the markets for Texas food and fiber products. The Center operates on a cooperative program of the Texas Agricultural Extension Service and the Texas Agricultural Experiment Station.

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SUPPLY-PRICE RELATIONSHIPS FOR TEXAS GRAPEFRUIT

Robert Branson and Carl Shafer*

The Research Purposes

Texas citrus production is increasing substantially as replanted groves in the Rio Grande Valley are entering bearing age. Extensive replanting was required following freezes in 1959 and 1961. Since that time there has also been a significant change in the volume of production from Florida as well as in the purchase behavior and size of the consumer market. Altogether, these circumstances have made it advisable to re-evaluate the supply-price relationships for Texas citrus. Consideration is given in this report only to Texas grapefruit. Similar market analysis research is desirable for Texas oranges, and therefore is being planned.

The purposes of the Texas grapefruit market analyses were several fold. In the first place it is helpful, in planning the marketing of Texas grapefruit, to have a knowledge of the principal factors which appear to influence the price for which the fruit can be sold. Such knowledge permits those marketing citrus to have a better conception of the likely price effects of any changes that do, or may, occur in these price influencing factors.

A second benefit of such market demand analyses is that the price effects of further increases in grapefruit production can be evaluated, if other things remain essentially unchanged. Thus decisions to make

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* Robert E. Branson, professor and Carl Shafer, associate professor, Department of Agricultural Economics and Sociology.

more plantings can be made under conditions of better knowledge of the potential price and income effects that can occur as a result. This is especially important for citrus fruit, inasmuch as a five to eight year period elapses before the trees reach anywhere near their full production.

Knowledge of the supply-price interrelationships, furthermore, can give a third advantage. There is always the question of how to best divide the fruit supply among fresh fruit and processed product markets, for the most profitable income from the crop.

Within certain limits, a study of supply-price relationships over recent years can also provide an indication of the price levels that can be anticipated for the season ahead, once the supply and demand factors are either reasonably well known or forecast. Any unanticipated changes in these supply and demand factors of course alter the price expectations. It is for this very reason that it is essential to pay careful and special attention to the basic assumptions behind any and all price estimates for a future time period.

Determination of the Factors Associated with the Price Level of Texas Grapefruit

Given the above advantages of knowing the relationship between Texas grapefruit prices and major price influencing market factors the first question was to find out what appear to be significant ones. Secondly, the degree of association between each and the price of grapefruit should be ascertained.

It is not too difiicult to list those factors which one would think likely affect the price of Texas grapefruit. That, however, is only the beginning. More important is to give each one an analytical test to learn if, in fact, some relationship to the fruit prices exists. Such

testing is a time consuming procedure, even though the basic statistical methods are known. Furthermore the association of changes in the factor and the fruit prices may be a one-to-one relationship or one that increases or decreases at a changing rate with changes in the factor concerned. Thus many experiments must be made before such questions can be answered.

Several sets of mathematical equations finally are constructed that represent those things that are related to the Texas grapefruit price and the degree of influence or association of each. This represents the midway point toward reaching the final target. It is necessary at this juncture to ask the price forecasting mathematical equation to make a price "forecast" for several recent years to see if it makes a good estimate of the actually realized fruit prices. Special attention is paid to recent years since it is the current or next marketing year for which the forecast information is really desired. Application of this test results in one or more equations performing better than the others. The best performing forecasting equation, on an all around basis, is selected for use.

Particular attention is paid to "all around" performance of a price forecasting model. In the case of Texas grapefruit, it is necessary to evaluate the behavior of a forecasting equation, or model, when the supply of fruit is increased substantially. Texas production may easily double over present levels. If the price forecasting model is properly constructed, it will still perform satisfactorily.

No price forecasting model should be used without its being continually reviewed and updated as each marketing season passes. Such a procedure keeps the data in the equation current. New food products, natural and synthetic, are making their appearance each year. Some have little and others much impact on the market for citrus fruits. By using as

current market data as possible, the effects of changing products, consumer buying behavior and other significant developments are allowed to have at least some record of their effects in the price forecasting model.

A list of the factors used in various price forecasting equations is given in the appendix to this report. Those variables that proved useful after careful testing are as follows:

1. A code variable (in this case zero-one) which signified whether or not a freeze had affected production of Texas citrus fruit during the year. A code of "1" was used for freeze years and one of "0" for all other years. The necessity for such a code was to separate freeze years from others since fruit price behavior differs substantially in such years.
2. Total season Texas grapefruit production in number of boxes of fruit.
3. The January 1 population of the 48 continental U.S. states.
4. Total season Florida production of grapefruit.
5. Supply in gallons frozen concentrated orange juice for the year. Total amount processed in the year is added to the carryover as of December 1.
6. Amount of disposable personal income.
7. The level of the Consumer Price Index for the year.

The foregoing variables, or factors, that were found to be closely associated, as a group, with Texas grapefruit prices, on tree, were reduced to a smaller number by means of a simple combining process which is generally used in such research. Instead of seven variables, there were five. Disposable personal income, factor #6, was combined with #7 to make disposable personal income already adjusted for general consumer price level changes.

Likewise, the population of the forty-eight states, factor #3, was combined with factor #2, #4 and #5 to give a separate figure of production of Texas grapefruit in boxes per 1,000 population, Florida production in boxes per 1,000 population and frozen orange juice in gallons per 1,000 population.

Instead of writing out all of these terms, a shorthand version is used in writing equations. When put in that form, the equation reads as shown below.

On-tree price for all Texas grapefruit.

(1) $P_{TA} = f(D, Q_{TF}, Q_{FF}, Q_{FZ})$

On-tree price for Texas grapefruit sold in the fresh fruit market.

(2) $P_{TF} = f(D, Q_{TF}, Q_{FF}, Q_{FZ})$

On-tree price for Texas grapefruit sold for processing.

(3) $P = f(D, Q_{TP}, Q_{FP}, Q_{FZ}, I)$

The shorthand terms represent the following list of factors in the equations.

P_{TA} = Price of all Texas grapefruit on-tree.

f = A mathematical designation that the left hand side of the equation is a function of, or dependent upon, all the variables listed in the parentheses.

D = A variable to signify whether the year was one in which freeze damage occurred to the fruit crop.

Q_{TA} = Quantity of total Texas grapefruit production divided by the 48 state U.S. population as of January 1 to obtain a figure of number of boxes of fruit per 1,000 population basis.

Q_{FZ} = Supply of frozen concentrated orange juice (amount processed during the crop year plus Dec. 1 carryover. This is reduced to gallons per 1,000 population, continental U.S.

I = Disposable personal income per capita deflated, or adjusted, by the Consumer Price Index.

F = When this is used in a subscript shorthand symbol, such as Q_{TF}, it means fruit for fresh market only is used.

P = Indicates that processing fruit only is concerned, when used as a subscript, as in Q_{TP}. When used alone as in equation (3) it stands for the on-tree price for grapefruit used for processing.

When the above equations are calculated, using data for 19 seasons, 1948/49 through 1966/67, a set of calculated values, known as coefficients, emerge for each factor in the equation. This coefficient indicates the relative weight that is assigned to the factor in conjunction with the size of units being used for the factor. The equations assume the following form.

For all Texas grapefruit:

$$(1)\quad P_{TA} = \underset{}{2.967} + \underset{(0.3819)}{1.1494}D - \underset{(0.00903)}{0.02148}Q_{TA} - \underset{(0.00423)}{0.00469}Q_{FA}$$

$$- \underset{(0.00122)}{0.00229}Q_{FZ} + \underset{(0.0078)}{0.00038}I$$

$$R^2 = 0.88$$

For Texas grapefruit sent to processing uses:

$$(3)\quad P_{TP} = \underset{}{0.9019} - \underset{(0.26823)}{0.28701}D - \underset{(0.01472)}{0.02370}Q_{TP} \quad \underset{(0.00576)}{0.00720}Q_{FP}$$

$$- \underset{(0.00104)}{0.00353}Q_{FZ} + \underset{(0.00051)}{0.00117}I$$

$$R^2 = 0.62$$

The figures in parentheses below each of the coefficients relates to what is known as the standard error of estimate value, which concerns the statistical significance of the factor in explaining the price level for the fruit. The R^2 value shown is a statistical term which denotes the proportion of the price changes in the 19 seasons analyzed that is accounted for by the equation. In other words, the equation for price forecasting for all Texas grapefruit, succeeded, on the average, in representing 84 percent of the price changes, whichever direction the actual market price moved.

In simple verbal terms, the equations can be stated in the following manner.

Price of all Texas grapefruit, on-tree	=	An intersect point of the demand curve on the graph	+	1.1494 times the existence of a freeze year		
- 0.02148 times the number of boxes of Texas grapefruit produced per 1,000 persons	=	0.00469 times the supply number of gallons of frozen orange juice per 1,000 persons	+	0.0038 times per capita disposable income adjusted for the consumer price level		

The equation for Texas grapefruit sold to the fresh market reads thusly:

Price of Texas fresh market grapefruit on-tree	=	5.0137 intersect point of demand curve on graph	+	1.0783 times the existence of a freeze year	-	0.0353 times the no. of boxes per 1,000 persons of Texas grapefruit sold to fresh market
	=					
- 0.01470 times the no. of boxes per 1,000 persons of Florida grapefruit sold to fresh market	=	0.00130 times the supply of frozen conc. orange juice for the yer per 1,000 persons	-	0.0048 times the per capita disposable income adjusted for the consumer price level		

For Texas grapefruit going to processing uses, the equation reads:

Price of Texas grapefruit going to processing uses	=	0.9019 intersect point of demand curve for graph	-	0.28701 times the existence of a freeze year	-	0.02370 times the no. of boxes per 1,000 persons of Texas grapefruit sold to processing uses
- 0.00720 times the no. of boxes per 1,000 persons of Florida grapefruit sold to processing uses	=	0.00353 times the year's supply of frozen concentrated orange juice for 1,000 persons	+	0.00117 times the per capita disposable income adjusted for the consumer price level.		

Interpretation of the Price Forecasting Equations

Whereas the foregoing, hopefully, gives some insight into the steps involved in developing a price forecasting model, the basic question is what does it mean to current marketing operations guidance? First, it is possible to determine, or forecast, the values that go into the equation for the market years concerned. Secondly, given the conditions of the preceding year and the change expected for the present, or coming, year a price forecast adjustment can be made from last season's actual prices. Let us consider each of these alternatives.

The forecast average price for a year can be determined by entering into the respective equations the values of the variables concerned. For example, if we want an all grapefruit season average price forecast, the values of the variables in that equation must be calculated and entered in that particular equation. Each variable, its source and how it is calculated are given in the order in which they appear in the equation.

Variable Factor	Source and Calculation
D	This represents an assumption for forecasting purposes of whether to expect a freeze or not. Of course, generally one is not anticipated.
Q_{TA}	Texas grapefruit production for the season must be estimated. Usually this estimate is taken from the crop forecast of the Crop Reporting Board of the U.S. Department of Agriculture. One may, of course, use their own forecast figures if there is disagreement with the early estimates

by the U.S.D.A. The first estimate from Washington is released on or about October 10.

The forecast production, which is in boxes, must then be divided by the population of the 48 continental states as estimated for January 1 of the marketing year, and then converted to the number of boxes per 1,000 persons. Population figures are reported monthly in the U.S. Survey of Current Business published by the U.S. Department of Commerce.

Q_{FA} Corresponding crop estimates for Florida grapefruit production are given by the Crop Reporting Board of the U.S.D.A. at the same time they are released for the Texas crop. The first official one is October 10. This production figure must likewise be reduced to the equivalent number of boxes for 1,000 population, using the same procedure as outlined for the preceding variable (Q_{TA}).

Q_{FZ} Here again it is necessary to obtain forecasts of the a) December 1 stocks of frozen concentrated orange juice and b) amount of frozen concentrated orange juice that will likely be processed during the coming season. Forecasts of these two figures usually can be obtained from the Triangle published by the Florida Citrus Mutual which are published weekly. The Florida Canners Association can also provide information, since it is a basic source of data. U.S. population data are again used to

reduce the forecast supply to a gallons per 1,000 persons basis.

I — Disposable personal income is reported monthly by the Survey of Current Business. September or October figures can be used, on an annual rate basis, or an annual average can be forecast for the marketing season. In the forecasting model, an estimate for the year was used. For example, the forecast for 1968-69 was based upon an estimate of disposable income for the year 1969.

Additional factors that appear in other equations are listed below:

TF — The quantity of Texas grapefruit that will be sold in the fresh market has to be estimated. This is done on the basis of past crop allocations, according to crop size, and in the light of recent developments that will likely affect past relationships. Data concerning the amount of a crop sold as fresh fruit and the amount processed are available from the Crop Reporting Board of the U.S. Department of Agriculture. These figures are also reported in several other sources such as Florida Agricultural Statistics Citrus Summary, which is issued annually by the Florida Department of Agriculture. Data are developed in cooperation with other governmental and industry agencies.

FF — The amount of Florida grapefruit that will likely go to fresh market has to be estimated

in the same manner as that for Texas grapefruit, discussed above.

TP — The quantity of Texas grapefruit to be processed is derived after determining the amount that will likely go to fresh market and subtracting the latter from the total crop estimate. Allowance can be made, if advisable, for abandoned fruit or other diversions from marketing channels.

FP — Florida grapefruit processing estimates are obtained by subtracting estimated fresh market supplies from the crop total. Allowance for now-market fruit, if any should be considered.

In preparing estimates of the proportion of a new crop that will be processed, it is advisable to intestigate the carryover stock position from the current year. Obviously, if supplies are heavy, the amount that will be processed will likely be less than otherwise would be the case. The better the information that can be brought to bear in establishing the quantities that go into the equations, the more likely they are to be accurate price forecasters.

Once the necessary information is developed and entered into the equation, the price forecast (1958 price level) evolves. The price is then multiplied by the current Consumer Price Index level, 122 for 1968. It must be clearly understood that the price forecast represents an estimate of the average price for the entire season. The value of fruit at any particular week will depend on within season factors rather than only those that affect the season as a whole. Furthermore, as changes become evident in forecasts of the size of the crop, of fruit quality for processing or for fresh market, changes in data entered into the price forecasting equation must be made. New or revised price forecasts are thus obtained.

Attention now is directed to an alternative view of short-range price forecasting. Perhaps the procedure suggested should be limited to use as only a short-cut approximation, for it essentially is that. Utilized in this approach is an adjustment of the average price for the previous season, or of the forecast price for the coming season, on a percentage basis as any one of the values of the factors in the estimating equation changes from the previously used values. Effects of changes in the variables can be summarized in table form as shown herewith,

Estimated Percent Change in Texas
All Grapefruit Price with
A One Percent Change in the Indicated Factors
(Assumes 1968-69 production level)

Factor	Percent change in price
Texas grapefruit production + 1% change	-0.96%
Florida grapefruit production, + 1% change	-1.57%
Supply of frozen concentrated orange juice + 1% change	-1.54%

The table is read as follows. If the production estimate of Texas grapefruit is increased by one percent, the price forecast should be lowered by 0.96 percent. Conversely, if the forecast is decreased by one percent, the price forecast is raised by 0.96 percent.

A one percent increase in Florida production of grapefruit is, on the average, associated with a 0.74 percent decline in the Texas fruit price.

Because of the extensive use and competitiveness of frozen concentrated orange juice as a breakfast fruit there is a reduction in grapefruit prices as the supply, and resultant use, of this orange juice increases. A one percent gain in frozen concentrated orange juice has been, on the average, associated with a decline of 0.52 percent in the on-tree price of Texas grapefruit, on an all fruit basis.

It is very important to realize, however, that the foregoing percentage relationships both can and do change over time. Especially as the size of the Texas grapefruit crop grows, such relationships will be altered. Again it is evident that a price forecasting model is never completed but must continually be updated.

Price changes for Texas grapefruit, on-tree, that goes to fresh market likewise can be summarized in percentage change terms.

Estimated Percentage Change in
Texas Fresh Market Grpaefruit,
On-Tree, with One Percent Change in
Indicated Factors
(Assumes 1968/69 production level)

Factor	Percent change in price
Texas fresh market grapefruit supply + 1% change	-1.29%
Florida fresh market grapefruit supply + 1% change	-1.13%
Supply of frozen concentrated orange juice + 1% change	-1.34%

Clearly the fresh fruit market for Texas grapefruit appears to be more price sensitive to changes in supply than is the total market (fresh and processed combined). A similar situation exists for most food products; therefore, this is as might be expected. Relative changes shown in the fresh fruit price table again have meaning only within a moderate range of the 1968-69 supply level. And, again, the percentages have to be kept up-to date over time.

The Price Forecasting Experience for the 1968-69 Marketing Season.

Quite obviously, a forecast of the price for Texas grapefruit is heavily dependent upon reasonably accurate estimates, a number of factors plus consideration of several more. The forecasting system is highly interdependent. Any time one factor changes so does the resulting forecast. Such is illustrated by the 1968-69 season experience. Therefore, it is pertinent to review that particular season.

The marketing season for Texas grapefruit begins usually in November and extends through April or May. As indicated elsewhere, the first official crop estimate for citrus is released by the Crop Reporting Board of the U.S. Department of Agriculture on October 10. Normally, the reports carrying the figures do not reach the recipients until about October 15-20. After the crop estimate was received, in October 1968 other necessary data required by the forecasting equation were secured or prepared. A price forecast was made on October 30, 1968 using the forecasting equations developed under Dr. Carl Shafer's supervision and direction. These equations were selected from approximately 200 that were developed and tested with the assistance of graduate students, during the four years of research conducted.

Development of the forecast involved the steps or inputs which are outlined herewith. Equations used were those previously discussed.

Price Forecast	Equation
All Texas grapefruit	$P_{TA} = 2.967 + 1.149D - 0.02148Q_{TA} - 0.00469Q_{FA} - 0.00229Q_{FZ} + 0.000\ 3I$ $R^2 = 0.$ [illegible]
Texas fresh market grapefruit	$P_{TF} = 3.9234 + 1.086D - 0.004123Q_{TF} - 0.011010Q_{FF} - 0.00177Q_{FZ}$ $R^2 = 0.88$
Texas processed grapefruit	$P_{TP} = 0.9019 - 0.28701D - 0.0237Q_{TP} - 0.0072Q_{FP} - 0.00353Q_{FZ} = 0.00117I$ $R^2 = 0.62$

Data requirements were developed for the equations as of late October 1968 for the 1968-69 marketing season.

a) Florida total grapefruit production estimated at 42 million boxes.

b) Texas total grapefruit production estimated at 7 million boxes.

c) Frozen concentrated orange juice supply estimated at 127.2 million gallons.

d) Disposable personal income per capita estimated at $2,475 (in 1958 equivalent dollars)

e) Consumer Price Index for September 1968 used which was 122.0.

f) January 1, 1969 population of the 48 continental U.S. States was estimated to be 203,395,000.

g) Texas grapefruit crop allocation used was at two alternative levels:

75% fresh and 25% processed
66% fresh and 34% processed

h) Florida grapefruit crop allocation estimated at :
41% fresh and 59% processed.

Resulting price forecasts were calculated and reported to the Texas Valley Citrus Committee during an official meeting of the full Committee during the month of November, 1968. The following was given.

	80 lb. box	ton
1. Texas all-grapefruit on-tree price forecast	$0.93	$23.25
2. Texas grapefruit fresh market, on-tree, price forecast.		
a. 75% fresh 25% processed	$1.00	$25.00
b. 66% fresh 34% processed	$1.14	$28.50
3. Texas grapefruit processing, on-tree, price forecast		
a. 75% fresh 25% processed	$0.62	$15.50
b. 66% fresh 34% processed	$0.53	$13.25

Following the above forecast presented at the November Texas Valley Citrus Committee meeting, a freeze occurred in Florida. At first, there appeared to be little Florida crop damage of consequence. Afterward, however, it was discovered that the yield of frozen concentrated orange juice was definitely affected.

Usually about 1.30 gallons of frozen concentrated orange is obtained per box of fruit. Instead the yield dropped to around 1.00 gallon per box.

In view of the freeze effects, a revised price forecast was requested by the Texas Valley Citrus Committee.

The new price forecasts were above those originally made because the Florida supply of frozen concentrated orange juice, in particular, was reduced to less than previous expectations. An average price, on-tree, for all Texas grapefruit was forecast at approximately $31.00 per ton,

The new price forecasts were above those originally made because the Florida supply of frozen concentrated orange juice, in particular, was reduced to less than previous expectations. An average price, on-tree, for all Texas grapefruit was forecast at approximately $31.00 per ton, or $6.00 above the previous forecast. Available data at the present time indicates that the average price received on-tree equivalent, for all Texas grapefruit (fresh and processed combined into an average and weighted by the volume to each market) was $34.00 per ton.

Whether the industry responded to our analysis indications that the total return would be more profitable by placing a smaller proportion of the crop in the fresh market, or for other reasons, such was done. Preliminary figures reveal that probably as much as 40 percent of the crop went to processing, instead of the 34 recommended. The more was in the right direction according to the market model the equations represent.

The full set of revised price forecasts were as follows:

	80 lb. box	ton
1. Texas all-grapefruit on-tree price forecast	1.24	$31.00
2. Texas grapefruit market, on tree, price forecast	1.43	35.75
3. Texas grapefruit, processed market, on-tree, forecast	.86	21.50

General Conclusions and Considerations

It would be coincidental indeed if the forecast price and the final price received for Texas grapefruit were the same. Any price forecasting has an error rante associated with it. A forecast of $31.00 per ton

(all fruit average on-tree) compared with a realized price of about $34.00 is considered as an acceptable forecast. One can readily expect the forecast to possibly be off as much as 10 to 15 percent. Industrial corporations that continually use market analysis forecasts recognize that the value of the value of the forecast is that it indicates in which part of the ball-park, so to speak, the play is likely to be made--right field, third base or home plate. Knowing this, a better management job of marketing can usually be planned and done.

Full consideration should be given to the fact that any forecasting model also assumes no more marketing effort than has been the average in the past. Inasmuch as this is the case, one could expect somewhat higher prices than the model forecasts if the industry does a better market development and promotion job. If the firms in the industry are sufficiently coordinating their marketing activities higher fruit prices possibly can be realized. Likewise, adverse circumstances may arise and reduce returns to lower levels.

Consideration, too, has to be given to the fact that the Florida grapefruit for 1968-69 had poorer exterior appearance than usual. At the same time, the Texas fruit looked better than is typical.

Promotion and establishment of a market for Texas grapefruit on the West Coast can also serve to raise prices above the average otherwise expected. The forecasting equations are based on a typical market distribution of Texas fruit over the 1949-68 period. That period, on the average, did not have much emphasis placed on California markets.

APPENDIX 1

PARTIAL LIST OF FACTORS TESTED IN CITRUS PRICE FORECASTING EQUATIONS

X_1 = Time

X_2 = Texas grapefruit average price on tree

X_3 = Texas fresh grapefruit average price on tree

X_4 = Texas process grapefruit average price on tree

X_5 = Texas grapefruit total production

X_6 = Texas grapefruit fresh market

X_7 = Texas grapefruit process market

X_8 = Florida grapefruit average price on tree

X_9 = Florida fresh grapefruit average price on tree

X_{10} = Florida process grapefruit average price on tree

X_{11} = Florida grapefruit fresh market

X_{12} = Florida grapefruit fresh market seedless

X_{13} = Florida grapefruit fresh market seeded

X_{14} = Florida grapefruit processed market

X_{15} = Florida grapefruit processed market seedless

X_{16} = Florida grapefruit processed market seeded

X_{17} = Arizona and California grapefruit production

X_{18} = U.S. population

X_{19} = Disposable personal income, per capita and deflated by the CPI

X_{20} = Consumer Price Index

X_{21} = U.S. Orange Production

X_{22} = U.S. frozen orange juice carryover

X_{23} = U.S. frozen orange juice pack

X_{24} = U.S. frozen orange juice total supply

X_{25} = Florida grapefruit total production

X_{26} = Ratio of Texas to Florida total production

X_{27} = Texas F.O.B. price per box

X_{28} = Florida F.O.B. price per box

$X_{29} = X_2 \div X_{20}$

$X_{30} = X_3 \div X_{20}$

$X_{31} = X_4 \div X_{20}$

$X_{32} = X_5 \div X_{18}$

$X_{33} = X_6 \div X_{18}$

$X_{34} = X_7 \div X_{18}$

$X_{35} = X_8 \div X_{20}$

$X_{36} = X_9 \div X_{20}$

$X_{37} = X_{10} \div X_{20}$

$X_{38} = X_{11} \div X_{18}$

$X_{39} = X_{12} \div X_{18}$

$X_{40} = X_{13} \div X_{18}$

$X_{41} = X_{14} \div X_{18}$

$X_{42} = X_{15} \div X_{18}$

$X_{43} = X_{16} \div X_{18}$

$X_{44} = X_{17} \div X_{18}$

$X_{45} = X_{21} \div X_{18}$

$X_{46} = X_{22} \div X_{18}$

$X_{47} = X_{23} \div X_{18}$

$X_{48} = X_{24} \div X_{18}$

$X_{49} = X_{25} \div X_{18}$

$X_{50} = X_{27} \div X_{20}$

$X_{51} = X_{28} \div X_{20}$